

ELECTROWEAK STRINGS PRODUCE BARYONS

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Abstract

We propose a new mechanism to generate baryons. Electroweak strings produced at the electroweak phase transition will introduce anomalous currents through interactions of the strings with the background electromagnetic field and through changes in the helicity of the string network. These anomalous currents will produce fluctuations in the baryon and lepton number. We show that in the two-Higgs model corrections to the action coming from the effective potential introduce a bias in the baryon asymmetry that leads to a net baryon number production.

One implication of the standard hot big bang cosmology is that from the relics of the early universe we should in principle expect the universe to contain the same abundance of baryons and antibaryons. However there is compelling empirical evidence that suggests the universe is made out of matter with a relatively small amount of antimatter. From the present asymmetry we can extrapolate that at the energy scale of the electroweak phase transition there was one part in 10^8 more matter than antimatter in the universe. The goal of baryogenesis is to explain this asymmetry between the amount of matter and antimatter.

Sakharov^[1] realized that there are three necessary conditions for any baryogenesis mechanism. First we need the existence of interactions that violate baryon number. The second condition is that the theory in consideration biases processes that violate baryon number towards the production of a net baryon number. This requires interactions that break the symmetries of charge conjugation (C) and the product of charge conjugation and parity (CP). Finally the interactions have to be in the presence of processes that are out of thermal equilibrium.

Recently it has been suggested that baryogenesis takes place at the electroweak phase transition (for a review see Refs.2,3and 4). The three main ingredients of this picture are:

First, in the electroweak model we know that in the field configuration space the sphaleron^[5] is the saddle point configuration connecting two different minima. These minima correspond to vacuum configurations with different baryon numbers. Transitions among the minima will change the baryon number.

Second, in order to produce a net baryon number asymmetry the electroweak interactions must violate C and CP . Extensions of the Minimal Standard Model (MSM) are required because the MSM violates C and CP in amounts too small to explain the baryon to antibaryon asymmetry.

Finally if the phase transition is first order, the bubbles produced will nucleate and expand. Near the expanding bubbles we shall have processes that depart from thermal equilibrium.

Different calculations of the characteristics of the phase transition^[7] seem to show that the transition is first order for small Higgs masses. The strength of the transition decreases as the Higgs mass increases to the point at which the transition is second order. In particular for the minimal standard electroweak model, the present lower bound of the Higgs mass (60 Gev) suggests that the phase transition is weakly first order. It also seems likely that in this range for the Higgs mass, the amount of anomalous baryon number violation after the phase transition will wash out any baryon number that was created during the phase transition. Finally we point out that if the transition is second order baryogenesis will not take place since the transition will be in thermal equilibrium.

Recently Brandenberger and Davis^[8] have proposed a new mechanism that uses strings produced at the electroweak phase transition. At the core of the strings we have trapped false vacuum in which transitions that change baryon number can occur through configurations that connect two vacua with different baryon number. The collapsing strings will be a source for the interactions that violate CP and of processes that are out of thermal equilibrium. In order for their mechanism to work the thickness of the string should be bigger than the magnetic screening length at high temperatures, so that the transitions that change baryon number can take place. This requires that the string core be thick so the Higgs mass is subsequently small. This implies that the phase transition will be more likely first order. Ref.9 also studies strings but these are produced at a symmetry breaking whose energy is slightly higher than the electroweak scale. The electroweak symmetry is unbroken at the core of the cosmic strings and transitions that change baryon number proceed as before.

In this paper we would like to review briefly electroweak strings (in particular Z-strings^[10]) and different mechanisms that imply magnetic fields at the time of the electroweak phase transition. We shall show how anomalous currents occur in the electroweak model. We shall examine the interaction of the magnetic Z-flux along the core of the string with the background magnetic field. This interaction will introduce baryon and lepton anomalous currents. Finally we shall study their implications as a possible mechanism for baryogenesis.

Ref.10 shows that in the standard electroweak model there are solutions that look like Nielsen-Olesen strings with magnetic Z-flux along their cores. Nambu^[10] has also shown that these strings can connect monopole-antimonopole pairs. The strings are metastable for some range in parameter space even though the standard electroweak theory is topologically trivial. However they are not stable for the physical value of the Weinberg angle, θ_w . We should expect that Z-string solutions survive for extensions of the standard electroweak model. For example Refs.11and 12 have shown that this is the case in the two-Higgs model extension. Ref.11 has also studied the stability of this solution to small perturbations. They find that for realistic values of the Higgs mass and the Weinberg angle the string is unstable.

Dvali and Senjanovic^[14]have also shown that if we consider the electroweak model with two Higgs doublets and we add an extra global $U(1)_{gl}$ symmetry to the theory, then there are topologically stable string solutions that also carry magnetic Z-flux along their core. The global $U(1)$ symmetry is obtained when we neglect the CP violating terms of the two-Higgs potential. These terms have the generic form $V_{CP} = \lambda(\Phi_1^\dagger\Phi_2 - \eta_1\eta_2 e^{i\theta})$, where θ is the CP violating phase. Therefore the only CP violation with these strings will come from the KM matrix and this has been shown to be too small to explain the baryon asymmetry. However these strings will be metastable for sufficiently small λ . This possibility is presently being investigated. Ref.15 shows that the minimal supergravity extension of the standard model has these topological solutions if the hidden sector has an exact R-symmetry. These strings have different characteristics than ‘ordinary’ embedded Z-strings produced at the time of the phase transition. We shall not consider them in this paper.

In what follows we will consider the embedded Z-strings solutions of Ref.10 . We shall assume extensions of the minimal standard model in which these strings are either stable or metastable. These strings will be stabilized by the nature of this baryogenesis mechanism. As we shall see, at the core of the strings massless baryons will be produced while outside they would be massive. Ref.13 has shown that bound states can improve the stability of nontopological solitons. However we should point out that no model has yet been found in which these strings are metastabe for realistic

values of the Higgs mass and the Weinberg angle.

Since the electroweak model has monopole solutions we expect that when our string network is produced we will have monopole-antimonopole ($M - \bar{M}$) pairs connected by strings and small string loops. The $M - \bar{M}$ and strings will interact with the background plasma and the monopoles will contract with some speed v_c . If l is the typical length of the strings in the network then after the time interval $\delta t_d = l/v_c$, the strings will decay. The strings will also have some transverse velocity with respect to the background that we will denote by v_t .

There are several mechanism that will generate primordial magnetic fields in the early universe. Some of them relay on physical effects during an inflationary phase transition^[16]. It has also been suggested^[17]that in the electroweak phase transition the fields have to be uncorrelated at distances larger than the initial correlation length(ξ). Therefore the gradients of the Higgs fields can not be compensated by the gauge fields for scales bigger than ξ . These gradients imply that we will have electromagnetic fields. Recently Ref.18 has used the Savvidy vacuum^[19]to generate magnetic fields at the scale of grand unification. They propose that magnetic field fluctuations at the GUT scales produce a phase transition to a new ground state with a non zero magnetic field. All these mechanisms imply strong magnetic fields at the time of the electroweak phase transition.

Now that we have reviewed electroweak strings and the existence of magnetic fields at the time of the phase transition, we shall show how the anomalous currents form in this model. In the electroweak model all the currents that couple with the gauge fields are free of anomalies so that the theory is renormalizable. However the baryon and lepton currents have anomalies of the form

$$\partial_\mu J_B^\mu = \partial_\mu J_L^\mu = N_f \left(\frac{g^2}{32\pi^2} W_a^{\mu\nu} \tilde{W}_{\mu\nu}^a - \frac{g'^2}{32\pi^2} Y_{\mu\nu} \tilde{Y}^{\mu\nu} \right). \quad (1)$$

Where N_f is the number of families, $W_a^{\mu\nu}$ $a = 1, 2, 3$ and $Y_{\mu\nu}$ are the $SU(2)$ and $U_Y(1)$ field strengths respectively, with the tilde referring to the usual definition of

the dual of the field strength. g and g' are the associated gauge couplings. Note that there is not an anomaly for the difference of the baryon to the lepton currents.

If we integrate both sides of Eq.(1) over a volume V and assume that the currents vanish at the surface of V we obtain that the baryon number $B = \int d^3x J_B^0$ changes in some time interval by,

$$\Delta B = \frac{N_f}{32\pi^2} \int dt \int d^3x \left(g^2 W_a^{\mu\nu} \tilde{W}_{\mu\nu}^a - g'^2 Y_{\mu\nu} \tilde{Y}^{\mu\nu} \right). \quad (2)$$

We see that if the r.h.s. of Eq.(2) is non zero we shall have a change in the baryon number. Another way to interpret this result is realizing that the integrand can be expressed as a total divergence so if we neglect surface effects and define the Chern-Simons numbers N_{cs} and n_{cs}

$$\begin{aligned} N_{cs} &= \frac{g^2}{32\pi^2} \int d^3x \epsilon^{ijk} \left(W_{ij}^a W_k^a - \frac{1}{3} g \epsilon_{abc} W_i^a W_j^b W_k^c \right) \\ n_{cs} &= \frac{g'^2}{32\pi^2} \int d^3x \epsilon^{ijk} Y_{ij} Y_k \end{aligned} \quad (3)$$

we can perform the time integral to obtain

$$\Delta B = N_f (\Delta N_{cs} - \Delta n_{cs}). \quad (4)$$

Therefore a change in the Chern-Simons number changes the baryon number. However the Chern-Simons number is not a meaningful physical quantity since it is gauge dependent and only changes in the Chern-Simons number will be gauge invariant.

After the phase transition the W_3^μ and Y^μ mix to form the massive Z^μ and the massless electromagnetic A^μ vector bosons.

In terms of these new vector fields Eq.(2) takes the form,

$$\begin{aligned} \Delta B &= \frac{N_f}{32\pi^2} \int d^4x [-g^2 \vec{E}_W^a \cdot \vec{B}_W^a + \alpha^2 \cos 2\theta_w \vec{E}_Z \cdot \vec{B}_Z + \\ &\quad \frac{\alpha^2}{2} \sin 2\theta_w (\vec{E}_A \cdot \vec{B}_Z + \vec{E}_Z \cdot \vec{B}_A) + i.t.] \end{aligned} \quad (5)$$

where $\alpha = \sqrt{g^2 + g'^2}$ and the electric and magnetic fields are defined respectively,

$$\begin{aligned} E_{\bar{a}W}^i &= W_{\bar{a}}^{0i}, & B_{\bar{a}W}^i &= \frac{1}{2}\epsilon_{jk}^i W_{\bar{a}}^{jk} \\ E_A^i &= A^{0i}, & B_A^i &= \frac{1}{2}\epsilon_{jk}^i A^{jk} \\ E_Z^i &= Z^{0i}, & B_Z^i &= \frac{1}{2}\epsilon_{jk}^i Z^{jk} \end{aligned} \quad (6)$$

and $\bar{a} = 1, 2$. The last term of Eq.(5)represents interaction terms of the electric and magnetic components of \vec{Z} and \vec{A} with $\vec{W}_{\bar{a}}$ and self-interactions of $\vec{W}_{\bar{a}}$. From Eq.(5)we see that there is no coupling between the electric and magnetic components of \vec{A} .

Once the W^1 , W^2 and Z bosons become massive they are exponentially suppressed unless there is some topological obstructions in their configurations. We will assume that after the phase transition we have Z -strings. At the core of the strings we have false vacuum and the Z bosons will be massless. Equation(5)can be reduced to

$$\Delta B = \frac{N_f}{32\pi^2} \int d^4x \left[\alpha^2 \cos 2\theta_w \vec{E}_Z \cdot \vec{B}_Z + \frac{\alpha^2}{2} \sin 2\theta_w (\vec{E}_A \cdot \vec{B}_Z + \vec{E}_Z \cdot \vec{B}_A) \right]. \quad (7)$$

From Eq.(7)we can infer a neat physical interpretation for the change in the baryon number. The first term in the r.h.s. of Eq.(7) is the change in the helicity of the string network^[20]. The second term can be easily understood if we realize that the background magnetic fields transform to electric fields in the frame of the moving strings. Therefore this term is the interaction of the magnetic Z flux along the core of the Z -strings with these electric fields. The last term will have the inverse interpretation of the previous one. The transformation of \vec{B}_Z from the string to the background frame will give us an \vec{E}_Z field that will interact with the background magnetic field.

To quantify the effect of the first term of Eq.(7)would require to know, using a computer simulation, the percentage of loops that are linked in the string network. We shall not do this calculation^[21]. In what follows we shall assume that the contribution from this term is not greater than the contributions coming from the other two terms.

Now that we have established a new mechanism to introduce anomalous currents that will violate baryon number, we need a process that biases these currents towards the production of a net baryon number. We are going to consider the two-Higgs model as an example. We shall show that the classical equations of motion for the strings are modified by the quantum corrections coming from the effective potential. For collapsing strings, these corrections will favor the production of baryons over antibaryons.

The relevant term in the effective action that biases the baryon number is given by the coupling of the relative phase θ of the two Higgs doublets with the W^a gauge bosons through the triangle diagram^[22]. For slowly varying θ this term can be expanded in a power series in the momenta^[23]. We also do the same diagram but replacing the W^a by the Y gauge fields. When we include both diagrams we find that the effective action contains the term,

$$\Delta S = \int d^4x \kappa \theta \frac{m_t^2}{T^2} \left(g^2 W_a^{\mu\nu} \tilde{W}_{\mu\nu}^a - g'^2 Y_{\mu\nu} Y^{\mu\nu} \right) \quad (8)$$

where $\kappa = \frac{14}{3\pi^2\zeta(3)}$ and the expansion is up to quadratic order in m_t/T . Using the standard definitions of Z^μ , A^μ vector bosons and assuming that the W^1 and W^2 are exponentially suppressed we obtain

$$\Delta S \simeq \frac{N_f \alpha^2}{64\pi^2} \kappa \sin 2\theta_w \frac{m_t^2}{T^2} \int d^4x \theta \left(\vec{E}_A \cdot \vec{B}_Z + \vec{E}_Z \cdot \vec{B}_A \right). \quad (9)$$

This term in the action violates C and P but conserves CP . However the evolution of θ will not conserve CP because of explicit breaking of CP in the two-Higgs potential. This term will modify the classical equations of motion of the string network. As the string network is formed Eq.(9) will bias the evolution of the interactions of the magnetic flux of the strings with the electromagnetic background. This will change the rate of baryon number produced. However in order to favor the production of a net baryon number we need that the angle θ changes in time in a definite direction, so that the transition rate will be more favorable towards the production of baryons over

antibaryons. The directionality of θ is given by the fact that the string network is collapsing. In particular we have seen that we will have strings bounded by monopole-antimonopole pairs and small loops that will be collapsing. At the moving ends of the strings, θ and the Higgs fields that describe the strings will evolve in a definite way in time.

Another way to understand this result is using the approach of Ref.22 . We can consider that θ is homogeneous in space and integrate by parts the first term in the r.h.s. of Eq.(8), we obtain

$$\Delta S \simeq \kappa' \frac{m_t^2}{T^2} \int dt \frac{d\theta}{dt} (N_{CS} - n_{CS}), \quad (10)$$

where $\kappa' = \frac{448}{3\zeta(3)}$. This corresponds to a potential that favors a change in the Chern-Simons number in a definite direction and because of (4) we will obtain a net change in the baryon number.

Now the only thing that is left to do is to show that Sakharov's third condition is also satisfied. We notice that the $M - \bar{M}$ pairs and string loops will collapse at relativistic speeds, this will give baryon production with processes that are out of thermal equilibrium.

Finally we are going to calculate a rough estimate of the amount of baryons produced. Unfortunately, we do not understand the formation of electroweak strings at the electroweak phase transition. There are some papers that study the statistical mechanics of strings and monopoles connected by strings^[24]. However they do not consider string interactions or break-up of strings into monopole antimonopole pairs, so that their results are not applicable to our problem. Also it seems that the Kibble mechanism will not apply. However topology has to play an important role in their formation since these strings are embedded defects. If the Higgs field winds non trivially around the vacuum manifold a string has to be produced if it is metastable. At present there is not a reliable method to estimate the density of strings or their length distribution. In order to obtain some numerical estimates we shall make the optimistic assumption that the density of strings is one per correlation volume.

The magnetic Z -flux along the core of the string is $\sim 2gT^2$. We also know that the strings will be moving in a background magnetic field. Refs.17 and 18 show that the energy density of the primordial magnetic fields scale as radiation, so that $B \sim gT^2$. The magnetic fields will be smooth in scales of the order of the correlation length $\xi \sim T$. At this temperature the scale of ξ corresponds to the inter-particle separation of the background plasma. As the correlation length grows the electromagnetic fields will start to be affected by the fact that the plasma is a very good conductor. So we would expect that the magnetic fields will be frozen in the plasma.

The transformation of these magnetic fields under a Lorentz transformation from the frame of the background plasma to the system of the string moving with velocity v_t with respect of the background is of the order of $E_A \sim \gamma v_t B_A$. As a first approximation we have neglected the electric fields assuming that the plasma is a very good conductor. The rate of baryon number produced per unit time and volume is

$$\Gamma \sim \frac{N_f}{64\pi^2} \alpha^2 \sin 2\theta_w E_A \cdot B_Z \sim \alpha_W^2 \gamma v_t T^4 \quad (11)$$

where $\alpha_W = g^2/4\pi \sim \alpha^2/4\pi$. This rate is three orders of magnitude higher than the rate of baryon number produced through thermal transitions across the sphaleron barrier ($\Gamma_s \sim \alpha_W^4 T^4$). This equation has to be modified to include the effect of corrections that come from the effective potential. As we saw before this contribution will be significant only at the end of the strings or in the collapsing loops. This will give that the volume in which the correction is effective is $\sim \delta^3$ where δ is the width of the string ($\delta \sim \lambda^{-1/2} \eta^{-1}$). The rate of baryon generation per string is

$$\frac{dN_B}{dt} \sim \alpha_W^2 \gamma v_t g^2 T^4 \epsilon \delta^3 \quad (12)$$

where ϵ is a dimensionless constant that gives the numerical contribution of Eq.(9) to the rate of baryon generation. To find the rate of increase in the baryon number density we have to multiply the previous expression by the density of strings. If we assume one string per correlation volume at the time of the phase transition

$(\xi^3 \sim \lambda^{-3}\eta^{-3})$, we obtain

$$\frac{dn_B}{dt} \sim \alpha_W^2 \gamma v_t g^2 \lambda^{1/2} \epsilon T^4. \quad (13)$$

This has to be integrated by the time that lasts the string network ($t_d \sim d/v_c \sim \lambda^{-1}\eta^{-1}$) so that

$$n_B \sim \alpha_W^2 \gamma v_t g^2 \lambda^{1/2} \epsilon T^3. \quad (14)$$

The entropy density at the time of the phase transition is

$$s = \frac{\pi^2}{45} \mu T^3, \quad (15)$$

where μ is the number of spin states. We obtain that the ratio between the baryon and entropy density is,

$$\frac{n_B}{s} = \frac{45}{\pi^2} \frac{1}{\mu} \alpha_W^2 \gamma v_t g^2 \lambda^{1/2} \epsilon. \quad (16)$$

For $\lambda \sim 1$ we need $v_t \epsilon \sim 10^{-4}$ in order to obtain the observational value of $n_B/s \sim 10^{-8}$.

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